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Inventory Model with Discontinuous Demand and Constant Deterioration

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ABSTRACT:
In this paper, an inventory model of deteriorated items has been developed with realistic assumption of discontinuous demand. The deterioration rate has been taken constant. Demand rate exists for fraction of total time cycle. It is considered that there is no demand during the remaining time.
Key-words: Discontinuous demand, deterioration and cycle time.

1 INTRODUCTION:
Several mathematical models for controlling the inventory have been developed by researchers. Some authors assumed the stock dependent demand rate while the others assumed the time dependent demand rate. Deterioration is an important factor in the discussion of inventory models. In most of the inventory models of deteriorating products it is assumed that the deterioration of the items starts as soon as they are brought to the inventory. Some authors assumed that the deterioration starts after some specific time which is called the life time. In the inventory models discussed so far the demand rate is assumed to be continuous whether constant or depending on current stock level or depending on time.
Covert, R.P. and Philip G.C. (1973) discussed **a**n EOQ model for items with Weibull distribution deterioration. **Donaldson (1977)** presented an inventory replenishment policy for a linear trend in demand. **Hari Kishan & Mishra, P.N. (1995)** discussed an **i**nventory model with stock dependent demand rate and constant rate of deterioration. **Hari Kishan & Mishra, P.N. (1997)** discussed an inventory model with exponential demand and constant rate of deterioration with shortage. **Sanjay Jain & Mukesh Kumar (2007)** discussed an inventory model with inventory level dependent demand rate, shortages and decrease in demand. **Hari Kishan, Megha Rani and Deep Shikha (2012)** discussed an inventory model of deteriorating products with life time under declining demand and permissible delay in payment.
But the demand rate is not continuous in practice. Shop opens for a certain period in a day during which demand rate may exist. After that period the demand does not exist while the deterioration of items continues till the inventory exists. Thus continuity of deterioration and discontinuity of demand is a realistic factor and consequently it is more fruitful to develop such models.
this paper, the inventory model of deteriorating products has been developed in which the deterioration rate is constant while the demand rate is assumed to be discontinuous function of time only.

2 Assumptions and Notations:

The inventory model has been developed on the basis of the following assumptions:
(i) Demand rate is the function of current time only
(ii) Replenishment is instantaneous.
(iii) Deterioration is constant.
The following notations are used in the model:
(i) q: The current stock level.
(ii) C: Set up cost for each new cycle.

(iii) C_i : Inventory carrying cost per unit per unit time.
(iv) C_d : The cost of deteriorated items.
(v) θ : The deterioration rate.
(vi) T: Cycle time.
(vii) K: The total average cost of the system.

3 Mathematical Model and Analysis:

Let Q be the initial stock level and q(t) be the stock level at time t. Let the demand exists up to the time t_1. Then the demand becomes zero due to the closing of the shop. Still due to the stock of items the deterioration continues up to time T. During this period, the stock level reduces due to deterioration and it becomes, say, S at time T. The model is represented by figure 1 given below:

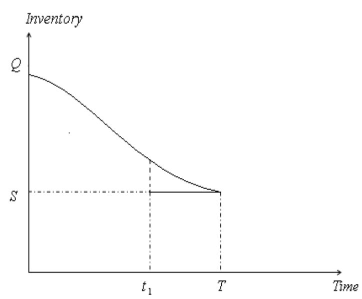

Figure 1

The mathematical form of this inventory model is given by

$$\frac{dq}{dt}+\theta q=-(at+b),\quad 0\le t\le t_1 \qquad \ldots(1)$$

$$\frac{dq}{dt}+\theta q=0,\qquad t_1\le t\le T \qquad \ldots(2)$$

The boundary conditions are

$$q(0)=Q, \qquad \ldots(3)$$

and $$q(T)=S. \qquad \ldots(4)$$

From equation (1) we have

$$qe^{\theta t}=-\int(at+b)e^{\theta t}dt+c$$

$$=-\frac{(at+b)}{\theta}e^{\theta t}+\frac{a}{\theta^2}e^{\theta t}+c. \qquad \ldots(5)$$

Using boundary condition (3) in (5), we get

$$c=Q+\frac{b}{\theta}-\frac{a}{\theta^2}.$$

From equation (1) we have

$$qe^{\theta t} = -\int (at+b)e^{\theta t}dt + c$$

$$= -\frac{(at+b)}{\theta}e^{\theta t} + \frac{a}{\theta^2}e^{\theta t} + c. \qquad \ldots(5)$$

Using boundary condition (3) in (5), we get

$$c = Q + \frac{b}{\theta} - \frac{a}{\theta^2}.$$

Putting this value of c in (5) we get

$$q = -\frac{(at+b)}{\theta} + \frac{a}{\theta^2} + (Q + \frac{b}{\theta} - \frac{a}{\theta^2})e^{-\theta t}. \qquad \ldots(6)$$

From equation (2), we have

$$qe^{\theta t} = c_1$$

or $$q = c_1 e^{-\theta t}. \qquad \ldots(7)$$

Using boundary condition (4), we have

$$S = c_1 e^{-\theta t}$$

or $$c_1 = Se^{\theta T}$$

$$\therefore q = Se^{\varpi(T-t)}. \qquad \ldots(8)$$

The remaining stock S is used in the next cycle.

Now the carrying cost during the cycle [0,T] is given by

$$C_1\left[\int_0^{t_1} qdt + \int_{t_1}^{T} qdt\right]$$

$$= C_1[\int_0^{t_1}\left(-\frac{(at+b)}{\theta} + \frac{a}{\theta^2} + \left(Q + \frac{b}{\theta} - \frac{a}{\theta^2}\right)\right)dt + \int_{t_1}^{T} Se^{\theta(T-t)}dt]$$

$$= C_1\left[[-\frac{\left(\frac{at^2}{2} + bt\right)}{\theta} + \left(Q + \frac{b}{\theta}\right)t]_0^{t_1} + \left[\frac{Se^{\theta(T-t)}}{-\theta}\right]_{t_1}^{T}\right]$$

$$= C_1\left[-\frac{at_1^2}{2\theta} + Qt_1 - \frac{S}{\theta} + \frac{S}{\theta}e^{\theta(T-t_1)}\right]. \qquad \ldots(9)$$

The deteriorating cost during the cycle [0,T] is given by

$$C_d\left[\int_0^{t_1}\theta qdt + \int_{t_1}^{T}\theta qdt\right]$$

$$= C_d\theta\left[-\frac{at_1^2}{2\theta} + Qt_1 - \frac{S}{\theta} + \frac{S}{\theta}e^{\theta(T-t_1)}\right]. \qquad \ldots(10)$$

The total average expected cost during the cycle [0,T] is given by

K= Setup Cost + Inventory Carrying Cost + Deterioration Cost

$$= \frac{C}{T} + \frac{C_1}{T}\left[-\frac{at_1^2}{2\theta} + Qt_1 - \frac{S}{\theta} + \frac{S}{\theta}e^{\theta(T-t_1)}\right]$$

$$+\frac{C_d\theta}{T}\left[-\frac{at_1^2}{2\theta}+Qt_1-\frac{S}{\theta}+\frac{S}{\theta}e^{\theta(T-t_1)}\right]$$

$$=\frac{C}{T}+\frac{(C_1+C_d\theta)}{T}\left[-\frac{at_1^2}{2\theta}+Qt_1-\frac{S}{\theta}+\frac{S}{\theta}e^{\theta(T-t_1)}\right].$$

Using the second approximation in the above expression, we get

$$K=\frac{C}{T}+\frac{(C_1+C_d\theta)}{T}\left[-\frac{at_1^2}{2\theta}+Qt_1+S(T-t_1)+S\theta(T-t_1)^2\right]. \quad \ldots(11)$$

Now we consider the following sub-cases:

Case I: When Q and T are fixed and t_1 is a variable.

In this case, the cost function K will be the function of single variable t_1. Therefore for minimum value of K, we have

$$\frac{dK}{dt_1}=0$$

or $$\frac{(C_1+C_d\theta)}{T}\left[-\frac{at_1}{\theta}+Q-S-S\theta(T-t_1)\right]=0$$

$$\Rightarrow \quad t_1\left(\frac{a}{\theta}-S\theta\right)=Q-S-S\theta T$$

$$\Rightarrow \quad t_1=\frac{(Q-S-S\theta T)}{\left(\frac{a}{\theta}-S\theta\right)}. \quad \ldots(12)$$

and $$\frac{d^2K}{dt_1^2}=\frac{(c_1+c_d\theta)}{T}\left[-\frac{a}{\theta}+S\theta\right]$$

>0 when $S\theta-\frac{a}{\theta}>0$.

Also t_1 will be positive when $S(1+\theta T)-Q>0$.

Thus K will be minimum if $S\theta-\frac{a}{\theta}>0$ and $S(1+\theta T)-Q>0$. This gives optimum value of t_1 as

$$t_1^*=\frac{(Q-S-S\theta T)}{\left(\frac{a}{\theta}-S\theta\right)}. \quad \ldots(13)$$

The optimum stock level at time $t=t_1^*$ is given by

$$q^*=-\frac{at_1^*}{\theta}+Q$$

$$=-\frac{a}{\theta}\left[\frac{S(1+\theta T)-Q}{S\theta-\frac{a}{\theta}}\right]+Q$$

$$=\frac{\left[S\theta^2 Q-aS(1+\theta T)\right]}{\left(S\theta^2-a\right)}. \qquad \ldots(14)$$

If $Q=\frac{a(1+\theta T)}{\theta^2}$ then q*=0. This shows that if $Q=\frac{a(1+\theta T)}{\theta^2}$ then there will be no stock at t_1^*. This gives minimum deterioration. Therefore this will be the most beneficial inventory. Thus optimal value of Q is given by

$$Q^*=\frac{a(1+\theta T)}{\theta^2}.$$

Case II: When Q and t_1 are fixed and T is variable.

In this case the cost function K will be the function of single variable T. Therefore for minimum value of K, we have

$$\frac{dK}{dT}=0$$

or
$$-\frac{C}{T^2}-\frac{(C_1+C_d\theta)}{T^2}\left[-\frac{at_1^2}{2\theta}+Qt_1+S(T-t_1)+S\theta(T-t_1)^2\right]$$
$$+\frac{(C_1+C_d\theta)}{T}\left[S+2S\theta(T-t_1)\right]=0$$

or
$$-C-(C_1+C_d\theta)\left[-\frac{at_1^2}{2\theta}+Qt_1+S(T-t_1)+S\theta(T-t_1)^2\right]$$
$$+(C_1+C_d\theta)\left[ST+2ST\theta(T-t_1)\right]=0$$

or
$$(C_1+C_d\theta)\left[S\theta\left(T^2-t_1^2\right)+\frac{at_1^2}{2\theta}-Qt_1+St_1\right]=C. \qquad \ldots(15)$$

This gives

$$T=\sqrt{\frac{1}{S\theta}\left[\frac{C}{(C_1+C_d\theta)}+S\theta t_1^2-\frac{at_1^2}{2\theta}+Qt_1-St_1\right]}. \qquad \ldots(16)$$

And
$$\frac{d^2K}{d^2T}=\frac{2C}{T^3}+\frac{2(C_1+C_d\theta)}{T^3}\left[-\frac{at_1^2}{2\theta}+Qt_1+S(T-t_1)+S\theta(T-t_1)^2\right]$$
$$-\frac{2(C_1+C_d\theta)}{T^2}\left[S+2S\theta(T-t_1)\right]+\frac{2S\theta(C_1+C_d\theta)}{T}$$
$$=\frac{2C}{T^3}+\frac{2(C_1+C_d\theta)}{T^3}\left[-\frac{at_1^2}{2\theta}+(Q-S)t_1+S\theta t_1^2\right]. \qquad \ldots(17)$$

Now if $S\theta-\frac{a}{2\theta}>0$ then obviously $\frac{d^2K}{d^2T}>0$. Therefore there will be minima for the cost function K.

Case III: When Q is fixed and T and t_1 are variables.

In this case, the cost function K will be the function of T and t_1. For the minimization of K we must have

$$\frac{\partial K}{\partial T}=0 \text{ and } \frac{\partial K}{\partial t_1}=0. \qquad \ldots(18)$$

And $$\frac{\partial^2 K}{\partial T^2}>0, \frac{\partial^2 K}{\partial t_1^2}>0 \text{ and } \left(\frac{\partial^2 K}{\partial T^2}\right)\left(\frac{\partial^2 K}{\partial t_1^2}\right)-\left(\frac{\partial^2 K}{\partial T\partial t_1}\right)^2>0. \qquad \ldots(19)$$

Differentiating (11) w.r.t. T and t_1 and putting them equal to zero, we get

$$-\frac{C}{T^2}-\frac{(C_1+C_d\theta)}{T^2}\left[-\frac{at_1^2}{2\theta}+Qt_1+S(T-t_1)+S\theta(T-t_1)^2\right]$$
$$+\frac{(C_1+C_d\theta)}{T}\left[S+2S\theta(T-t_1)\right]=0 \qquad \ldots(20)$$

or $$-C-(C_1+C_d\theta)\left[-\frac{at_1^2}{2\theta}+Qt_1+S(T-t_1)+S\theta(T-t_1)^2\right]$$
$$+(C_1+C_d\theta)\left[ST+2ST\theta(T-t_1)\right]=0,$$

or $$-C+(C_1+C_d\theta)\left[\frac{at_1^2}{2\theta}-Qt_1+St_1+S\theta T^2-S\theta t_1^{\,2}\right]=0. \qquad \ldots(21)$$

And $$\frac{(C_1+C_d\theta)}{T}\left[-\frac{at_1}{\theta}+Q-S-2S\theta(T-t_1)\right]=0 \qquad \ldots(22)$$

or $$t_1\left(\frac{a}{\theta}-2S\theta\right)=Q-S-2S\theta T. \qquad \ldots(23)$$

Solving the equations (21) and (23), the values of t_1 and T can be obtained. These values may be denoted by $t_1^{\bullet}$ and $T^{\bullet}$.

Differentiating (11) twice with respect to t_1 and T, we get the values of $\frac{\partial^2 K}{\partial t_1^2}, \frac{\partial^2 K}{\partial t_1\partial T}$ and $\frac{\partial^2 K}{\partial T^2}$. It can be shown that for the values of $t_1^{\bullet}$ and $T^{\bullet}$, these derivatives satisfy the relations

$\frac{\partial^2 K}{\partial T^2}>0, \frac{\partial^2 K}{\partial t_1^2}>0$ and $\left(\frac{\partial^2 K}{\partial T^2}\right)\left(\frac{\partial^2 K}{\partial t_1^2}\right)-\left(\frac{\partial^2 K}{\partial T\partial t_1}\right)^2>0$. Thus the values $t_1^{\bullet}$ and $T^{\bullet}$ are the optimal values. Using these values of $t_1^{\bullet}$ and $T^{\bullet}$, we can obtained the optimal values of the other parameters such as minimum value of K , the cost function.

4 CONCLUSIONS:

In this paper, the inventory model of deteriorating products has been developed in which the deterioration rate is constant while the demand rate is assumed to be discontinuous function of time only. The cycle time is taken as T. This case has been discussed in three sub cases. This model can further be extended for other forms of demand rate, variable deterioration rate and life time.

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